specific switch type but much of the information could be used in designing a normally closed switch or a three-position switch using a cross polarization pick-off to retrieve the power dissipated in resistance sheets. Many of the design principles discussed apply equally to more general types of amplitude modulators.

Appendix

Because of the high frequencies and small sizes involved in the 70-kmc region the ordinary resistance sheet made by evaporating metal film on a dielectric backing sheet causes loss to the wave whose polarization is such that the wave is ordinarily considered to be unaffected. This loss arises in the following fashion. A dielectric sheet stretched across dominant mode waveguide so that its plane is everywhere perpendicular to the plane of the dominant mode electric vector of the empty guide causes components of electric field to occur parallel to the sheet in the altered dominant mode configuration. These field components along the dielectric will cause current flow and hence loss if one or both sides of the dielectric are coated with a metal film. If the sheet is centered on a diametral plane of a round cylindrical waveguide the tangential field components vanish on the diametral plane itself so this is clearly the place to put the evaporated metal film. Accordingly, the resistance sheets used in the switch are made by evaporating metal on one dielectric sheet and covering this with a second dielectric sheet of equal thickness.

The propagation constant of a rectangular waveguide containing a dielectric sheet coated with a film of conductivity σ ohms^{-1} on each side has been obtained by solving the appropriate boundary value problem and this solution indicates the magnitude of the loss expected in the actual circular waveguide. The rectangular guide has a height b, the dielectric thickness is c, and the dielectric center plane lies on the center plane $(b/2)$ of the guide. If the attenuation constant α is small compared to the free space phase constant β_0, and if c is small compared to b, the attenuation constant and phase constant β can be expressed simply:

$$\alpha = 377 \frac{c^2}{2b} \beta_0^2 \sigma \left(\frac{\epsilon_r - 1}{\epsilon_r}\right)^2,$$

$$\beta = \beta_0 \left[1 - \frac{\pi^2}{a^2\beta_0^2} + \frac{c}{b}\left(\frac{\epsilon_r - 1}{\epsilon_r}\right)\right],$$

where ϵ_r is the relative dielectric constant of the dielectric sheet and a is the width of the waveguide. The attenuation per wavelength becomes appreciable at very high frequencies only because c, the dielectric thickness, cannot be scaled down indefinitely. Experimentally, we have found that a coating of 0.01 ohm^{-1} on a 0.0035-inch piece of Nylar ($\epsilon_r = 3.7$) in 0.150-inch round guide causes a loss of 0.75 db per inch at 70 kmc. The theoretical prediction from the above equation is 0.625 db per inch in 0.148-inch square waveguide. If the conducting film is centered in the dielectric sheet, as mentioned earlier, symmetry requires that there be no conduction loss since the electric field is everywhere normal to the film and the film is much less than a skin depth in thickness. Experimentally, a film constructed in this fashion produces less than 0.1 db per inch of loss in the orthogonally polarized wave.

Theoretical Analysis of the Operation of the Field-Displacement Ferrite Isolator*

KENNETH J. BUTTON†

Summary—**A theoretical analysis of the resistance-sheet isolator is carried out, and numerical solutions are obtained for the forward and reverse propagation constants of the distorted dominant mode in a rectangular waveguide containing a transversely magnetized thick ferrite slab displaced slightly from the side wall. The microwave electric field patterns within the waveguide are plotted for several values of the physical design parameters of the isolator for which experimental performance data have been reported. Field patterns are used to describe the principles of the isolator and to select the optimum values of slab thickness, internal dc magnetic field, ferrite magnetization, and location of the slab in the waveguide for the idealized isolator. Evidence is presented to show that it is necessary to use a comparatively thick ferrite slab located in a very small usable range of distances from the side wall. The appropriate value of internal dc magnetic field is simply related to the magnetization of the ferrite and to the frequency. It has not been necessary to take into account the perturbing effects of the resistance card or matching techniques in order to explain the basic design principles.**

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INTRODUCTION

THE nonreciprocal distortion of the dominant-mode electromagnetic fields in a waveguide containing a magnetized ferrite is now a well-known phenomenon. The different RF electric and magnetic field patterns for forward and reverse propagation in a ferrite-loaded rectangular guide were demonstrated theoretically by Lax, Button, and Roth,[1] and the nonreciprocal field-distortion principle has been used to build resistance-sheet isolators in rectangular guide by Fox, Miller, and Weiss[2] and others,[3] and in a circular guide by Melchor, Ayres, and Vartanian.[4] In each case, isolation was achieved by placing a vane of resistance material in a region of the guide where the RF electric intensity was large for the reverse direction of propagation and small for the forward direction. Recently, Weisbaum and Seidel[5] have reported the experimental performance and a theoretical treatment of a 6200-mc field-displacement isolator consisting of a rectangular waveguide containing a thick ferrite slab transversely magnetized and located nearly against the side wall of the guide. These experimental investigations, particularly the latter,[5] have established that the principal features of the field-displacement isolators are: large reverse-to-forward ratio of attenuation (>150 to 1 in db at 6200 mc), very low loss for the forward direction of propagation (~0.2 db), adequate bandwidth (>8%), low values of applied dc magnetic field (a few hundred oersteds) and moderate, but probably not high, power handling capacity.

The purpose of the present investigation is to establish the theoretical basis for the choice of design parameters for a field-displacement isolator of the type shown in Fig. 1. The notable successes of the laboratory models have depended largely upon the experimenters' ability to visualize the different RF electric field patterns of the dominant mode for the reverse and forward propagation. Additional improvement of performance was obtained by successive variation of parameters. The following exact solutions of the ferrite-loaded waveguide problem will demonstrate the method for finding electric field patterns and will illustrate the patterns in several cases for which experimental performance data have been reported.[5] This paper does not take into account the small perturbations due to the introduction of the resistance card or matching techniques, so that some experimental adjustments will still have to be made.

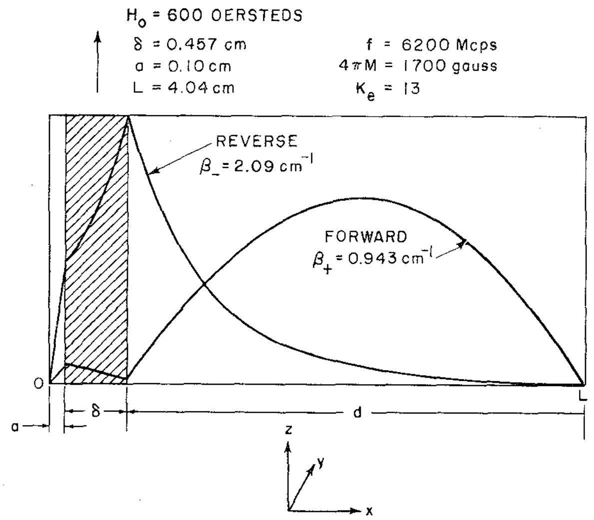

Fig. 1—Transversely magnetized ferrite slab in rectangular waveguide showing the calculated microwave electric field patterns for reverse and forward propagation that are appropriate for a field-displacement isolator. A resistance card may be placed on the right-hand face of the slab to provide attenuation of the reverse wave where the electric intensity is maximum.

Weisbaum and Seidel[5] have already treated thoroughly the theoretical aspects of the shape, location, and resistivity of the resistance card, and they have also discussed the influence of the height of the ferrite slab on performance, matching, and choice of parameters. Furthermore, they have taken into account the possible existence of a longitudinal component of the microwave electric field.

THE BOUNDARY VALUE PROBLEM

The problem of the infinite rectangular waveguide containing a transversely magnetized ferrite slab was formulated by Kales, Chait, and Sakiotis[6] and the procedures for solving the transcendental equations have been fully described by Lax, Button, and Roth[1] in their discussion of the thin-slab ferrite phase shifter. Since the same procedures and notation are used to obtain the following results except in the discussion of the significance of the effective ferrite permeability: $1/\rho = \mu_{eff}/\mu_0$, this paper will be restricted to the presentation and interpretation of results with only the following skeleton description of procedure to bring in the necessary definitions.

The RF electric field for the distorted dominant mode in a rectangular guide containing a transversely magnetized ferrite slab (Fig. 1) may be expressed as

[1] B. Lax, K. J. Button, and L. M. Roth, "Ferrite phase shifters in rectangular waveguide," *J. Appl. Phys.*, vol. 25, pp. 1413–1421; November, 1954.

[2] A. G. Fox, S. E. Miller, and M. T. Weiss, "Behavior and applications of ferrites in the microwave region," *Bell Sys. Tech. J.*, vol. 34, pp. 5–103; January, 1955.

[3] E. H. Turner, "Field displacement isolators at 55 kmc," IRE TRANS. ON ANTENNAS AND PROPAGATION, vol. AP-4, pp. 583–584; July, 1956.

S. Weisbaum and H. Boyet, "A double-slab ferrite field displacement isolator at 11 kmc," PROC. IRE, vol. 44, pp. 554–555; April, 1956.

[4] J. L. Melchor, W. P. Ayres, and P. H. Vartanian, "Energy concentration effects in ferrite loaded wave guides," *J. Appl. Phys.*, vol. 27, pp. 72–77; January, 1956.

[5] S. Weisbaum and H. Seidel, "The field-displacement isolator," *Bell Sys. Tech. J.*, vol. 35, pp. 877–898; July, 1956.

S. Weisbaum and H. Boyet, "Field displacement isolators at 4, 6, 11 and 24 kmc," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-5, pp. 194–198; July, 1957.

[6] M. L. Kales, H. N. Chait, and N. G. Sakiotis, "A nonreciprocal microwave component," *J. Appl. Phys.*, vol. 24, pp. 816–817; June, 1953.

$$E_z = (A \sin k_a x) \exp[-j\beta y], \quad \text{region } a$$
$$E_z = [B \sin k_a(L - x)] \exp[-j\beta y], \quad \text{region } d$$
$$E_z = (C \exp[jk_m x] + D \exp[-jkm]) \exp[-j\beta y], \quad \text{region } \delta \qquad (1)$$

where β is the propagation constant (the attenuation constant is neglected),[7] $k_a^2 = \omega^2/c^2 - \beta^2$, $k_m^2 = \omega^2 \epsilon \mu_{\text{eff}} - \beta^2$, and $\epsilon/\epsilon_0 = K_e$, the dielectric constant. The transverse wave number in regions a and d is denoted by k_a and in region δ by k_m. The effective permeability

$$\mu_{\text{eff}}/\mu_0 = \frac{(1 + \chi_{xx})^2 + \chi_{xy}^2}{1 + \chi_{xx}} \qquad (2)$$

may be positive or negative depending upon the relative magnitudes of χ_{xx}, the diagonal component of the susceptibility tensor, and χ_{xy}, the off-diagonal component.[8] The E fields of (1) are substituted into Maxwell's equations to obtain the components of the RF magnetic field h_x and h_y in each region and the boundary conditions are then applied to obtain four homogeneous simultaneous linear equations for the amplitude coefficients A, B, C, and D whose secular determinant must vanish. B is taken equal to unity for convenience. The determinantal equation which must be solved exactly by numerical methods has the form[1]

$$a = \frac{L - \delta}{2} - \frac{1}{2k_a} \cos^{-1}[P(\pm\beta)] \qquad (3)$$

where $P(\pm\beta)$ is given in the Appendix. The solution was accomplished by substituting trial values of β into $P(\pm\beta)$ and computing separate values for the location of the slab, a, for each sign of β.

Location of the Ferrite Slab

The numerical solutions of (3) yield a graph of $\beta_\pm$ vs a for constant values of all other physical parameters. β_+ and β_- are the phase constants for forward and reverse propagation, respectively. Solutions for three different values of the internal dc magnetic field, H_0, are shown in Fig. 2. It is possible to visualize the RF electric field patterns by reading the values of β from Fig. 2 for a particular value of a. For values of β above the horizontal line, k_a is imaginary because β^2 is larger than ω^2/c^2 in the relation $k_a^2 = \omega^2/c^2 - \beta^2$. In that case the mode has a hyperbolic sine dependence in the empty region of the guide and may actually be plotted in region d simply by tracing out the function $\sin k_a(L - x)$. For example, the values of $\beta_\pm$ used in plotting the fields of Fig. 1 were taken from the 600-oersted curves of Fig. 2 for $a = 0.1$ cm. Here k_a^+ is real and k_a^- is imaginary. If k_a^+ were just equal to π/d, the sine curve of Fig. 1 would go to zero at the face of the slab and this is the condition for minimum forward attenuation in the resistance card. To achieve this exactly, the slab must be moved closer to the wall (see Fig. 3) but this would sharply reduce β_- for the reverse direction of propagation and k_a^- would become real. Eventually, for $a < 0.05$ cm, the fields would be nearly identical for both directions of propagation as shown in the inset of Fig. 2 and there would be virtually no isolation. Therefore, although it is easy to define the conditions for which the forward-propagating field goes nearly to zero at the face of the slab, this is not a sufficient condition for an isolator.

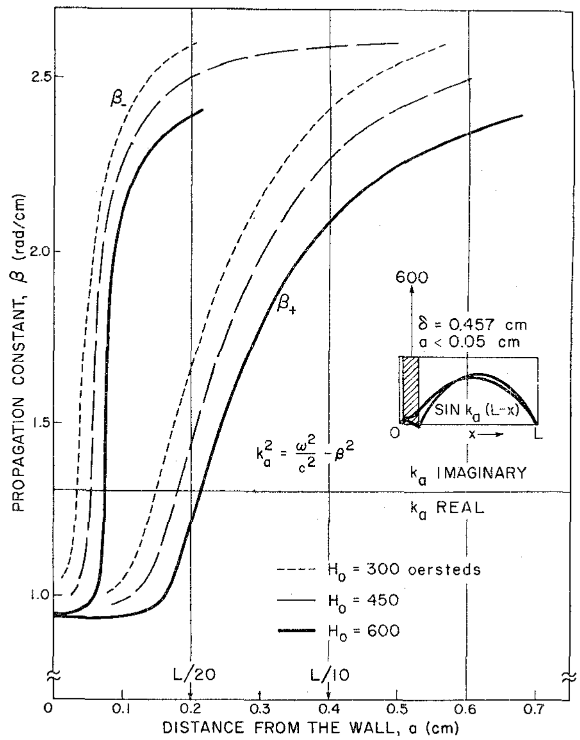

Fig. 2—Solutions of (3) showing the phase constant vs the distance of the slab from the wall with the internal dc magnetic field, H_0, as a parameter. The fields of Fig. 1 were computed by using the values of β from the 600-oersted curves at $a = 0.1$ cm. The fields shown in the inset for very small values of a will not provide sufficient isolation.

The most critical design parameter appears to be the location of the slab. It would be desirable, for example, to place the slab far from the wall to get the largest value of β_- and thus increase the amplitude of the spike in the reverse field pattern. If, however, the slab is placed too far from the wall, β_+ rises sharply for the forward propagation, k_a^+ decreases toward imaginary values, and the electric intensity at the face of the slab rises rapidly, resulting in increased forward attenuation which cannot be tolerated. Even in this relatively large waveguide (4 cm wide) the range of freedom in placing the slab is restricted to about 1 mm in order to satisfy both necessary conditions shown in Fig. 1—a sharp

[7] K. J. Button, "Theory of Ferrites in Rectangular Waveguide," Lincoln Lab., M.I.T., Lexington, Mass., Quart. Prog. Rep. on Solid-State Res., pp. 55–56; November, 1955.

[8] In terms of the Polder permeability tensor components, $1 + \chi_{xx} = \mu$ and $\chi_{xy} = -j\kappa$. See D. Polder, "On the theory of ferromagnetic resonance," *Phil. Mag.*, vol. 40, pp. 99–115; 1949.

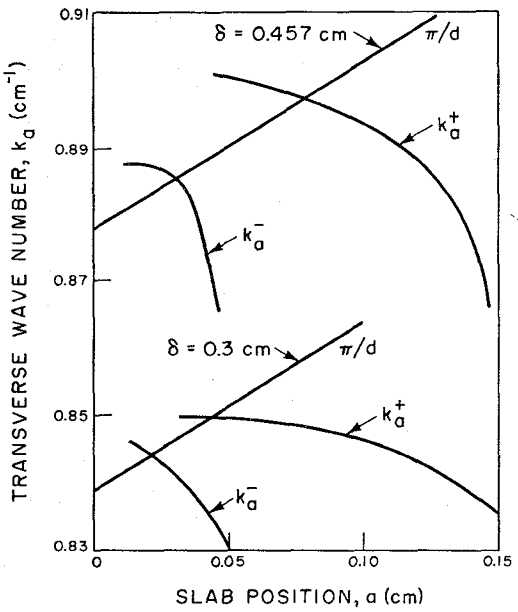

Fig. 3—Plots of the transverse wave number k_a^+ for forward propagation and k_a^- for the reverse direction. The straight lines show the values of π/d for two different slab thicknesses. The intersection of the curves with the straight lines gives the value of a for which the E field goes to zero at the face of the slab. These have been plotted for $H_0=600$ oersteds (negative permeability) because there is no intersection for smaller fields (positive permeability).

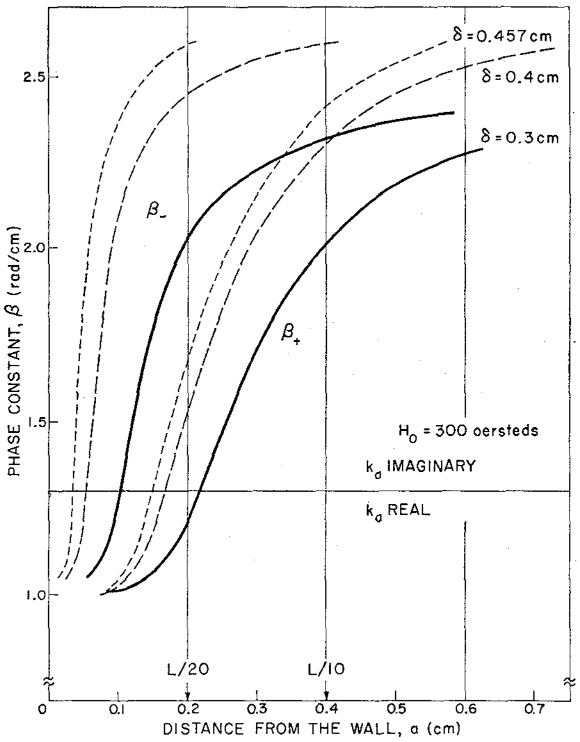

Fig. 4—Solutions of (3) with the slab thickness as a parameter. The result is not sensitive to changes in slab thickness if the slab is sufficiently thick.

hyperbolic spike in the E field for reverse propagation, and a near-zero E field at the face of the slab for forward propagation. The practical range of location is further restricted because β_- should be very large in order both to avoid the steepest portion of the β_- curve (Fig. 2) and to obtain the highest possible spike in the reverse E field.

Slab Thickness

Fig. 4 shows solutions for three different slab thicknesses at an internal dc magnetic field intensity of 300 oersteds. The value of δ is not critical provided that the slab is "thick," that is, about 10 per cent of the waveguide width or greater. The 3-mm case that is shown indicates, however, that curves for thinner slabs fall away rapidly both from high values of β and from small values of a. It is much more difficult to find a value of a for the thinner slab cases where β_+ will be sufficiently small and where β_- will also be very large.

Some selected field patterns are shown in Fig. 5, opposite. The fields of Fig. 5(a) have been plotted for a thick slab with the internal dc magnetic field as a parameter. This shows that the reverse attenuation for the thick slab case is not highly sensitive to changes in magnetic field. If the slab thickness is decreased, however [Fig. 5(b)], the reverse electric field intensity near the ferrite edge begins to collapse. If, in addition, the magnetic field is increased to 600 oersteds within the thin slab, Fig. 5(c) shows that both forward and reverse electric fields become similar and the problem has been reduced to that which has been solved previously for the phase shifter.[1]

DC Magnetic Field and Ferrite Magnetization

An ideal isolator design requires: 1) an hyperbolic sine dependence of the electric field for reverse propagation, and 2) a sinusoidal dependence for forward propagation, which in particular, is near zero at the face of the slab where the resistance card is to be located. The first condition can be assured by using a thick slab slightly spaced from the wall with nearly any convenient dc magnetic field below resonance.

Some care must be taken to achieve the second condition. Weisbaum and Seidel[5] have shown analytically that the forward E field cannot be brought to zero at the face of the slab unless the effective permeability (2) is negative. If the ferrite is completely saturated magnetically and the saturation magnetization is known accurately, then this condition of negative effective permeability may be selected from a plot like that of Fig. 6. When the ferrite is not saturated, the net magnetization is lower than the saturation value and the point of zero permeability shifts to higher fields as indicated by Fig. 6.

The value of a for which the forward E field goes to zero may be found, if desired, by solving (3) and plotting a curve like that of Fig. 2. Alternatively, the condition $k_a=\pi/(L-a-\delta)$ may be imposed on (3) to yield

$$a = \frac{1}{k_a}\tan^{-1}\left[\frac{-k_a\dfrac{\mu_{\text{eff}}}{\mu_0}}{-\dfrac{j\beta}{\theta}+k_m\cot k_m\delta}\right] \qquad (4)$$

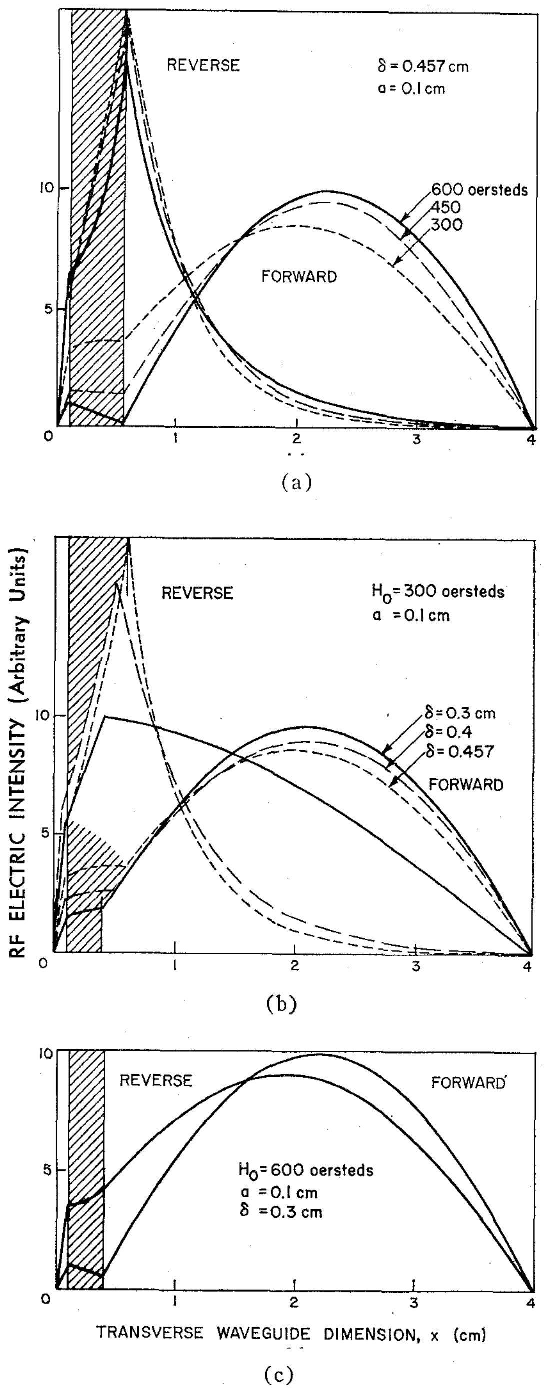

Fig. 5—Selected E-field patterns for (a) three different values of H_0, the internal dc magnetic field intensity, (b) three different slab thicknesses, and (c) a slab that is too thin for effective isolator performance. All calculations have been performed for 6200 mc in standard C-band guide.

where $\theta=(1+\chi_{xx})/\chi_{xy}$. Then this can be solved in the region of negative effective permeability by substituting successive trial values of β until a value of a is obtained which satisfies the relation $L=a+\delta+d$. This latter method has been used successfully in similar problems.[9]

[9] B. Lax and K. J. Button, "Theory of new ferrite modes in rectangular wave guide," *J. Appl. Phys.*, vol. 26, pp. 1184–1185; September, 1955.
K. J. Button and B. Lax, "Theory of ferrites in rectangular waveguides," IRE Trans. on Antennas and Propagation, vol. AP-4, pp. 531–537; July, 1956.

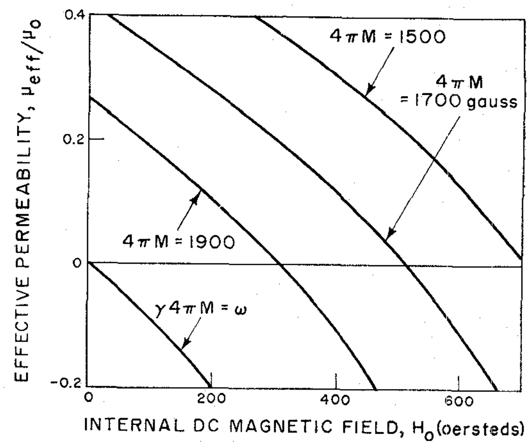

Fig. 6—Effective permeability at 6200 mc vs internal dc field H_0 for several values of magnetization. The magnetization for the lowest curve satisfies the relation $\gamma 4\pi M=\omega$ which may be used as a guide in selecting the appropriate magnetization for a given operating frequency. The curves have been carried back to zero field for comparison purposes but the magnetization would be a function of H at very low fields.

Low-Field Loss

At lower microwave frequencies, the problem of "low-field loss" associated with unsaturated ferrite material is an important consideration and often cannot be avoided. This loss should be nonreciprocal because of the strong energy concentration and rejection effects in field-distortion devices and its influence must be considered at any operating frequency if the applied dc field is small. It may not be assumed from a study of the E fields alone that a great deal of magnetic loss in the ferrite can be tolerated. For the reverse direction of propagation under the conditions shown in Fig. 1, both the electric and magnetic portions of the RF energy are propagated almost entirely within the ferrite so the low-field loss effects should increase the reverse attenuation. If the forward-propagating RF energy were excluded from the ferrite, the performance would be improved by magnetic losses. However, in the forward direction the RF magnetic field is not entirely outside of the ferrite as the electric field is. The transverse component h_x^+ is just β_+ times the E field in the ferrite and is therefore excluded. But the longitudinal component h_y^+ is slightly larger than h_{0y}, the y component of the empty waveguide h field. It is to be expected then, that at very small values of applied dc field, magnetic loss acting on the propagating energy through h_y^+ would moderately increase the forward attenuation. The experimental data[5] show that the loss at very low values of applied field is nonreciprocal, but the magnetization was too small to interpret the effect of low-field loss quantitatively. The designer may be able to turn magnetic losses to his advantage, especially at low microwave frequencies, by choosing the saturation magnetization large enough (Fig. 6) to provide a region of negative effective permeability at small values of applied field where part of the material is not saturated (and the magnetization is less than the saturation value). Negative effective permeability makes it possible to minimize the forward-propagating energy within the ferrite which is a necessary condition for minimum forward attenuation both in the ferrite and in the resistance card.

Conclusion

It has been shown that the dominant-mode RF electric field configurations for both directions of propagation may be used to choose the design parameters for the resistance-sheet isolator. It has been concluded that a thick ferrite slab nearly against the side wall operated in a region of negative effective permeability will give the best performance. These are the same conditions used in the laboratory[5] with the possible exception of the condition on effective permeability.

The condition $\gamma 4\pi M \approx \omega$ (from Fig. 6) requires that the ferrite magnetization be known fairly accurately at the temperature and dc field intensity that are to be used. Then a value of internal dc field (corrected for demagnetization) may be chosen for operation in the region of negative effective permeability.

The uncertainties in the values of $4\pi M$ and H_0 will require the experimental adjustment of the applied field in order 1) to operate far enough below resonance, 2) to minimize forward loss over the band, 3) to find a conveniently small applied field, and 4) to nearly saturate the ferrite.

Appendix

The transcendental expression of (3) is somewhat complicated and must be solved numerically. The function $P(\pm\beta)$ from Lax, Button, and Roth[1] is

$$P(\pm\beta) = -\frac{pr \pm q(p^2+q^2-r^2)^{1/2}}{p^2+q^2} \tag{5}$$

where

$$p = \frac{1}{2}\left(\frac{k_a{}^2\mu_{\text{eff}}{}^2}{\mu_0{}^2} + \frac{\beta^2}{\theta^2} - k_m{}^2\right)$$

$$q = j\,\frac{\beta k_a \mu_{\text{eff}}}{\theta\mu_0}$$

$$r = \frac{1}{2}\left(\frac{k_a{}^2\mu_{\text{eff}}{}^2}{\mu_0{}^2} - \frac{\beta^2}{\theta^2} + k_m{}^2\right)\cos k_a(L-\delta) + k_m \cot(k_m\delta)\,\frac{k_a\mu_{\text{eff}}}{\mu_0}\sin k_a(L-\delta).$$

Acknowledgment

The author is greatly indebted to Dr. Benjamin Lax for several helpful discussions and for his suggestion of essential points that have been incorporated in this paper. He also wishes to thank Dr. Gerald S. Heller for his criticism of the theory, Richard N. Brown for his assistance with the exploratory computations, and Mrs. Billie H. Houghton for computation of the final data.

Reciprocity Relationships for Gyrotropic Media*

R. F. HARRINGTON† AND A. T. VILLENEUVE†

Summary—**Reversal of the dc magnetic field in gyrotropic media transposes the tensor permeability and permittivity. It is shown that this also transposes the impedance, admittance, and scattering matrices of any device. It follows from this that the usual reciprocity statements for isotropic media apply to gyrotropic media if one reverses the dc magnetic field whenever an interchange of source and measurer is made.**

Introduction

Ferrites and gaseous plasma have been called "nonreciprocal" media because the usual reciprocity theorem[1] does not apply to them. However, a modified reciprocity theorem, stated by Rumsey and attributed to M. H. Cohen,[2] applies to such media. A number of useful and interesting interpretations of this reciprocity theorem are presented in this paper.

A ferrite in a dc magnetic field is characterized, insofar as an ac field is concerned, by a tensor permeability $[\mu]$ and a scalar permittivity ϵ.[3] Both $[\mu]$ and ϵ are independent of the amplitude of the ac field so long as it is sufficiently small. The $[\mu]$ is transposed if the dc magnetic field is reversed. A gaseous plasma in a dc magnetic field is characterized, insofar as an ac field is concerned, by a tensor permittivity $[\epsilon]$ and a scalar permeability μ.[4] Both $[\epsilon]$ and μ are independent of the amplitude of the ac field so long as it is sufficiently small. The $[\epsilon]$ is transposed if the dc magnetic field is reversed. The term *gyrotropic* is used to denote a medium characterized by $[\epsilon]$ and $[\mu]$, independent of the amplitude of an

* Manuscript received by the PGMTT, December 23, 1957; revised manuscript received, January 29, 1958.

† Dept. of Elec. Eng., Syracuse University, Syracuse, N. Y.

[1] S. A. Schelkunoff, "Electromagnetic Waves," McGraw-Hill Book Co., Inc., New York, N. Y., p. 478; 1943.

[2] V. H. Rumsey, "The reaction concept in electromagnetic theory," *Phys. Rev.*, vol. 94, pp. 1483–1491; June 15, 1954. Errata, vol. 95, p. 1705; September 15, 1954.

[3] D. Polder, "On the theory of ferromagnetic resonance," *Phil. Mag.*, vol. 40, pp. 99–115; January, 1949.

[4] H. Suhl and L. R. Walker, "Topics in guided wave propagation through gyrotropic media, part I," *Bell Sys. Tech. J.*, vol. 33, pp. 579–659; May, 1954.